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# PROBABILITY THEORY WITH SIMULATIONS

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Abstract  
Contents  
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This is an introductory textbook to probability theory and statistics with the usual material taught at most universities.

Its special feature, however, is that it contains interactive simulation files. These files are important, because the real life meaning of most of the notions of probability theory and statistics can be experienced only if we make a large number of experiments, not only once, but several times, and not only under a given set of conditions, but under modified conditions, as well.

The simulation files included in this textbook make it possible that the reader could see the results of many experiments, and could repeat them several times, and he or she could modify the parameters of the problem, as well.

Since the simulation files are written in Excel, students themselves can easily construct similar simulation files. Their activity will increase their confidence and interest in the subject.

The book consists of five parts:

1. Probability of events
2. Discrete distributions
3. Continuous distributions in one-dimension
4. Two-dimensional continuous distributions
5. Statistics

The author is devoted to write an exercise-book soon, which will - hopefully - help the students to learn not only the probabilistic and statistic notions but the necessary Excel tricks to construct simulation files according to their own needs.

Key words and phrases: Probability, Random number, Random variable, Discrete distribution, Continuous distribution, Expected value, Statistics, Regression, Confidence interval, Hypothesis test.

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