

MIKLÓS FERENCZI – MIKLÓS SZŐTS

# MATHEMATICAL LOGIC FOR APPLICATIONS

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Abstract  
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This book is recommended for those readers who have completed some introductory course in Logic. It can be used from the level MSc. It is recommended also to specialists who wish to apply Logic: software engineers, computer scientists, physicists, mathematicians, philosophers, linguists, etc. Our aim is to give a survey of Logic, from the abstract level to the applications, with an emphasis on the latter one. An extensive list of references is attached. As regards problems or proofs, for the lack of space, we refer the reader to the literature, in general. We do not go into the details of those areas of Logic which are bordering with some other discipline, e.g., formal languages, algorithm theory, database theory, logic design, artificial intelligence, etc. We hope that the book helps the reader to get a comprehensive impression on Logic and guide him or her towards selecting some specialization.

**Key words and phrases:** Mathematical logic, Symbolic logic, Formal languages, Model theory, Proof theory, Non-classical logics, Algebraic logic, Logic programming, Complexity theory, Knowledge based systems, Automated theorem proving, Logic in computer science, Program verification and specification.

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Károly Varasdi

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